**Linear Algebra. Midterm. Variant 2.**

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| First name | Last name | Group | Points#1 |
|  |  | BS1- |  |

I am, \_\_\_\_\_ (initials), confirming that I have read the following rules and agree to comply with them, that all solutions on this paper is my own work.

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (signature)

Rules:

* no talking AT ALL is allowed during the exam and after it (if you are still in the room)
* when time is up, you have to put down your pen (pencil) and do NOT write anything else
* you can NOT leave your seat till the end of the test
* any electronic devices are not allowed

1. True of false (with a reason or a counterexample)? (3 points)
   1. *A* and *AT* have the same number of pivots.
   2. *A* and *AT* have the same dimension of Nullspaces.
   3. If the Row space equals Column space, then *A* = *AT*

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| First name | Last name | Group | Points#2 |
|  |  | BS1- |  |

1. Find the best straight-line fit (Least squares) to the measurements (4 points)

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Then find the projection matrix of vector onto the column space of

 (3 points)

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1. Find an orthonormal basis for the subspace spanned by the vectors: and (3 points). Then express in the form of (2 points).

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1. Find the dimensions of the four fundamental subspaces associated with , depending on the parameters and . (5 points)

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1. Find a vector ***x*** orthogonal to the Row space of , and a vector ***y*** orthogonal to the Column space, and a vector ***z*** orthogonal to the Nullspace:

. (5 points)

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1. Project the vector ***b*** = (1; 2) onto the lines through the vectors ***a1*** = (0; 1) and ***a2*** = (1; 1) (2 points). Draw the projections ***p1*** and ***p2*** and ***p1 + p2*** (1 point). Explain why ***p1 + p2***is not equal to ***b***. (2 points)

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1. Find a polynomial (with real coefficients) for which:

(\* 5 points \*)